TABLE 1. COMPARISON OF EXPLICIT APPROXIMATIONS TO COLEBROOK'S FRICTION FACTOR EQUATION

	Moody Eq. 2	Wood Eq. 3	Jain Eq. 5	Churchill Eq. 6	Chen Eq. 9	Zigrang & Sylvester Eq. 11	Zigrang & Sylvester Eq. 12
Average Absolute Deviation, %	4.3	2.7	0.68	0.65	0.11	0.15	0.017
Maximum Absolute Deviation, %	16.0	6.0	3.10	3.10	0.32	0.95	0.110

numerical solution of Eq. 1 was conducted. A matrix of 60 test points was formed combining each of 10 roughness ratios with six different values for Reynold's number. The roughness ratios were 4×10^{-5}, 5×10^{-5}, 2×10^{-4}, 6×10^{-4}, 1.5×10^{-3}, 4×10^{-3}, 8×10^{-3}, 1.5×10^{-2}, 3×10^{-2} and 5×10^{-2}. The Reynold's numbers were 4×10^3, 3×10^4, 10^5, 10^6, 10^7 and 10^8. The absolute deviations relative to Colebrooks' equation were computed from

$$E = \left| \frac{f_D - f_{DC}}{f_{DC}} \right| \tag{13}$$

and accumulated over the sixty points calculated for each of the seven explicit equations. The results are shown in Table 1.

Although each of the explicit approximations given in Eqs. 9, 11 and 12 is adequate for computational purposes, Eqs. 11 and 12 are recommended. Equation 9 requires more effort than Eq. 11 but less effort than Eq. 12. Likewise, Eq. 9 is more precise than Eq. 11 but less precise than Eq. 12. Consequently, Eq. 11 is recommended for use with hand-held calculators because it is relatively simple for its degree of precision with respect to the Colebrook equation equation. Clearly, Eq. 12 should be used with programmable calculators and digital computers.

NOTATION

D = inside diameter of pipe
E = Error, defined by Eq. 13
ϵ = Roughness height
f_D = Darcy friction factor
f_{DC} = Darcy friction factor calculated from the Colebrook equation
Re = Reynolds number

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Direct Contact Heat Transfer with Change of Phase: Theoretical Model

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The economical water desalination technique has led to increased interest in research on direct-contact heat transfer between two immiscible liquids. The mechanism of heat transfer during the course of vaporization of single liquid drop in an immiscible liquid medium has not yet been clarified.

Sideman and Isenberg (1967) presented a model about the quasi-steady state heat transfer, outside a two-phase bubble moving in a potential flow field, supposing that the heat flows into the bubble exclusively through the liquid-liquid interface situated at the bottom. The unsatisfactory agreement between the theoretical and experimental values of these authors is due to the assumption of having neglegible inside thermal resistance of the bubble.

The rigid sphere model of Tochitani et al. (1977) agrees quantitatively with their experimental results. This gives a more reasonable value of heat transfer coefficients than those obtained by the expression given by Sideman and Taitel (1964). The higher value of heat transfer coefficients given by the rigid sphere model during early stages of evaporation is ascribed to the predominant resistance prevailing inside the two-phase bubble. It is thus clear that the previous authors have not considered the proper configuration of liquid-liquid interface for transfer of heat.

The aim of this paper is to develop a theoretical model based on actual liquid-liquid heat transfer interface, by introducing the effect of viscous shear on the spreading of dispersed liquid over the bubble surface.

Considering only the surface and interfacial free energies, Mori (1978) obtained the following relation for the film spreading coefficient in terms of interfacial tensions:

$$S_d = \sigma_c - (\sigma_d + \sigma_{dc})$$

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Mori (1978) and Selecki and Gradon (1972) discuss the importance of the surface tension of both liquids, which cause two completely different mechanisms of evaporation. According to these authors the configuration of the type as shown by regular line in Figure 1 is favored for the systems having a positive value for S_d. Under the influence of the gravitational field, most of the mass of the dispersed liquid will accumulate at the bottom of the two-phase bubble, while the residual mass may spread over the upper surface in the form of a thin film, depending on the value of spreading coefficient. Based on the force balance over the bubble surface, the equilibrium location of the end of this film is estimated. For theoretical analysis, the following assumptions are introduced:

1. The two-phase bubble is considered special and it rises rectilinearly with a velocity U.
2. The film of dispersed liquid is thin enough so that there is no circulation inside the film.
3. The flow around a bubble obeys the Stokes theory.

For a thin film, the effect of gravity can be neglected as compared with that of viscous shear exerted on the film surface. Using these assumptions, the force balance in the θ direction is described by the following relation:

$$\frac{3}{2}\frac{\mu_c U}{R}\sin\theta = -\frac{1}{R}\frac{d\sigma}{d\theta} \tag{1}$$

It is reasonably presumed that the film is so formed that the surface pressure vanishes at $\theta = \alpha$ because there the film meets the continuous liquid-gas interface with an interfacial tension σ_c. Therefore, the boundary condition at $\theta = \alpha$ is given by,

$$\sigma|_{\theta=\alpha} = \sigma_c \tag{1a}$$

The other boundary condition is assumed as,

$$\sigma|_{\theta=\beta} = \sigma_c - S_d \tag{1b}$$

Integrating Eq. 1 using Eqs. 1a and 1b, we obtain,

$$1.5\frac{\mu_c U}{S_d} = \frac{1}{\cos\alpha - \cos\beta} \tag{2}$$

It is presumed that heat transfer to two-phase bubble takes place through the thin film lying between the zenithal angles β and α. Further this film being very thin, it is reasonable to assume that it offers negligible inside resistance. The energy equation in spherical co-ordinates is expressed as:

$$RU\frac{\partial T}{\partial r} + u_\theta\frac{\partial T}{\partial\theta} = R\alpha_c\frac{\partial^2 T}{\partial r^2} \tag{3}$$

According to assumption of Tochitani et al. (1977) that the thermal boundary layer around a two-phase bubble is much thinner than either the hydrodynamic boundary layer or the bubble radius, the tangential velocity can be approximated as:

$$u_\theta = \dot{u}(r - R) \tag{4}$$

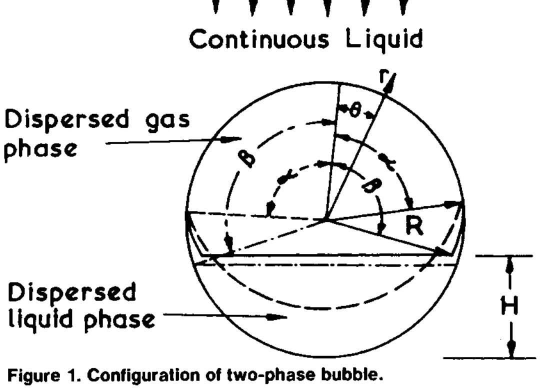

Figure 1. Configuration of two-phase bubble.

Models	Symbols
Eqn (15)	—
Tochitani Eqn (16)	- - -
Sideman Eqn (17)	- · — · -

The stream function $\psi = \int_R^r Ru\theta \sin\theta dr = \frac{1}{2}(r - R)^2\dot{u}R\sin\theta$. To simplify, let $Y = (2\psi)^{1/2}$, then:

$$Y = (r - R)(\dot{u}R\sin\theta)^{1/2} \tag{5}$$

Replacing r by Y as a variable in Eq. 3, we get,

$$\frac{Y}{(\dot{u}R^5\sin^3\theta)^{1/2}}\frac{\partial T}{\partial\theta} = \alpha_c\frac{\partial^2 T}{\partial Y^2} \tag{6}$$

Let

$$X = \int_\alpha^\beta (\dot{u}R^5\sin^3\theta)^{1/2}d\theta \tag{7}$$

Therefore Eq. 6 reduces to

$$Y\frac{\partial T}{\partial X} = \alpha_c\frac{\partial^2 T}{\partial Y^2} \tag{8}$$

The boundary conditions for Eq. 8 in terms of X and Y are:

$$T = T_\infty:\ Y > 0,\quad X < 0$$
$$T = T_\infty:\ Y = \infty,\quad X \geqslant 0$$
$$T = T_L:\ Y = 0,\quad X \geqslant 0$$

The solution of Eq. 8 with the above boundary conditions takes the form:

$$\frac{T_\infty - T}{T_\infty - T_L} = 1 - \frac{1}{0.893}\int_0^Z e^{-Z^3dZ} \tag{9}$$

where

$$Z = Y(9X\alpha_c)^{-1/3} \tag{10}$$

The local heat flux $q\theta$ is given by,

$$q\theta = K_c\left(\frac{\partial T}{\partial r}\right)_{r=R} = K_c\left(\frac{\partial T}{\partial Z}\cdot\frac{\partial Z}{\partial Y}\cdot\frac{\partial Y}{\partial r}\right)_{r=R}$$

With the help of Eqs. 5, 9 and 10 the heat flux $q\theta$ takes the form:

$$q\theta = K_c\left(\frac{T_\infty - T_L}{0.893}\right)\left(1 - \frac{Y^3}{9X\alpha_c}\right)(9X\alpha_c)^{-1/3}(\dot{u}R\sin\theta)^{1/2} \tag{11}$$

To simplify, the series term $(1 - Y^3/9X\alpha_c$ is evaluated corresponding to 3.5 wt. %, vapor in the vicinity of which various experimental runs for different systems covering an initial drop size form 1 to 3.5 mm and the temperature difference between the two immiscible liquids from 1 to 12.2°C, give a high value of heat transfer coefficient. The rigidity of the two-phase bubble is well maintained at this stage. Replacing θ by α and using the average value of the series term for various runs, we have:

$$h_{c\theta} = \frac{q_\theta}{\Delta T} = 0.58104\,K_c(X\alpha_c)^{-1/3}(\dot{u}R\sin\theta)^{1/2} \tag{12}$$

The average outside heat transfer coefficient related to the total area of a two-phase bubble is calculated as:

$$h_c = \frac{1}{4\pi R^2}\int_\alpha^\beta h_{c\theta}2\pi R^2\sin\theta\,d\theta \tag{13}$$

Assuming the surface of a two-phase bubble is rigid, and the flow around it obeys the Stokes theory, we have:

$$\dot{u} = 1.5\frac{U\sin\theta}{R} \tag{14}$$

Using Eqs. 12, 7 and 14 in 13, we have,

$$h_c = 0.314\frac{K_c}{R}\left[\frac{RU}{\alpha_c}\right]^{1/3}\left[\beta - \alpha - \frac{(\mathrm{Sin}2\beta - \mathrm{Sin}2\alpha)}{2}\right]^{2/3} \tag{15}$$

If the term $Y^3/9X\alpha$ in Eq. 11 is neglected, the value of the constant becomes 0.291 which is same as obtained by Tochitani et al. (1977) in their model:

$$h_c = 0.291\,K_c\alpha_c^{-1/3}R^{-2/3}U^{1/2}\left[\pi - \alpha + \frac{\mathrm{Sin}2\alpha}{2}\right]^{2/3} \tag{16}$$

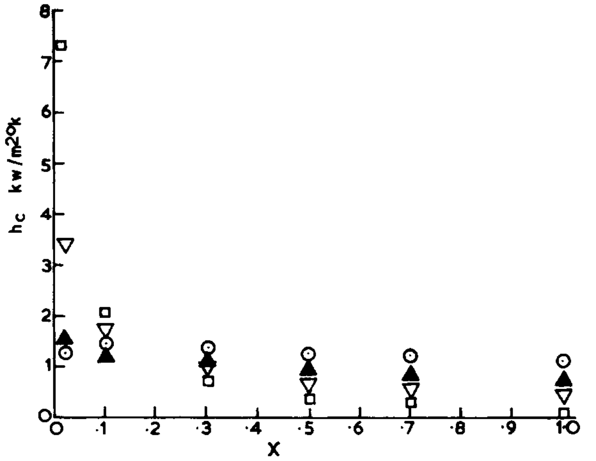

Fig. 2 Comparison of different models for heat transfer coefficient for pentane-water system.

Models	Symbols
Eqn (15)	▲
Tochitani Eqn (16)	▽
Sideman Eqn (17)	□
Sideman Run No 14	⊙

The value of h_c obtained by Sideman and Isenberg (1967) has the form:

$$h_c = K_c\left(\frac{3U_\infty}{2\pi R\alpha_c}\right)^{1/2}\left(\mathrm{Cos}\beta - \frac{1}{3}\mathrm{Cos}^3\beta + \frac{2}{3}\right)^{1/2} \qquad (17)$$

The mass balance for constant mass bubble Figure 1 yields:

$$H^2(3R - H) = \frac{4(mR_o^3 - R^3)}{m - 1} \qquad (18)$$

or in terms of the weight percent vapor content x, we have:

$$\frac{R^3}{R_o^3} = 1 + x(m - 1) \qquad (19)$$

where, $H = R(1 + \mathrm{Cos}\beta)$ is now time dependent. (20)
Lumping the last three equations together we get,

$$(3\,\mathrm{Cos}\beta - \mathrm{Cos}^3\beta + 2) = \frac{4(1 - x)}{1 + x(m - 1)} \qquad (21)$$

For various values of angle β in Eq. 21, we get the corresponding values of x. The experimental data of Sideman and Taitel (1964) for the Pentane-water system has been used for investigation. Knowing the values of x, the values of velocity and radius can be calculated with the aid of Eq. 19 and the experimental expressions. The value of α is then caloulated from Eq. 2. The values thus obtained are substituted in Eqs. 15, 16 and 17 to get the theoretical values of h_c. The experimental data for heat transfer coefficients are calculated on similar lines as carried out by Sideman and Taitel (1964). From Figure 2, it is clear that our model gives values that are very close to the experimental values as compared to those of Tochitani et al. (1977) and Sideman and Isenberg (1967).

The higher values of h_c obtained by Tochitani et al. (1977) during the early stages of evaporation are due to the higher contribution of the angle term at the lower values of α and the reverse is the case towards the later stages of evaporation.

NOTATION

K = thermal conductivity
m = ratio of liquid to vapor density of the bubble
r = radial distance in spherical co-ordinates
R = radius of two-phase bubble
R_o = initial radius of the drop of dispersed phase
T = temperature
T_L = interface temperature between continuous and dispersed phases
S = spreading coefficient
T_∞ = bulk temperature
U = rise velocity of two-phase bubble
$\dot{u}$ = velocity gradient
u_r = tangential velocity
x = weight percent vapor
μ = dynamic viscosity
σ = tension at interface
α_c = thermal diffusivity

Subscript

c = continuous phase
d = dispersed phase

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An Improved Single Particle Char Gasification Model

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Reactions between porous solids and gases are usually interpreted in terms of idealized models based on sharply defined reaction zones within the particle. For example the widely employed homogeneous and shrinking core models represent the extremes that might be encountered in the limits of very rapid and very slow diffusion of the reacting gases. Srinivas and Amundson (1980) recently proposed a model for the gasification of a single char particle in the presence of steam, hydrogen, carbon monoxide,

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